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A Consideration on hcp ^{3}He in the Paramagnetic Phase

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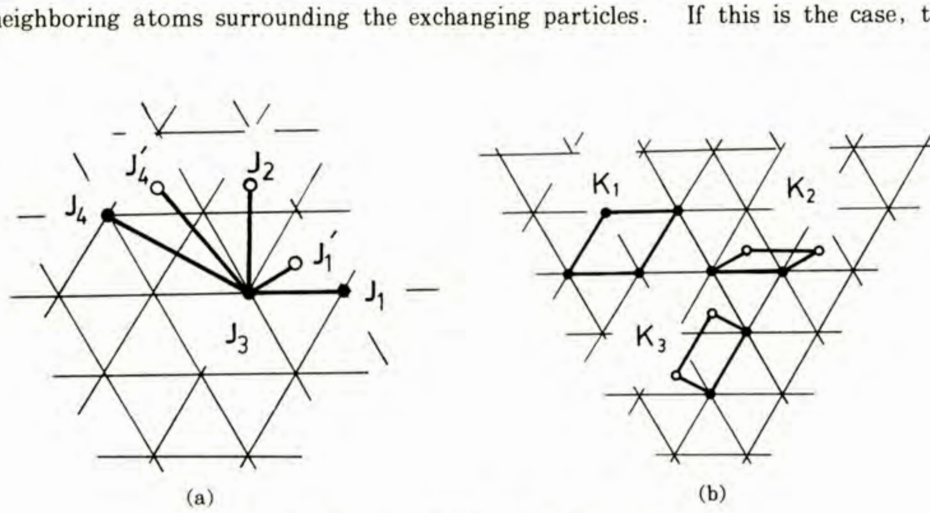
Abstract

Magnetism of hcp ^{3}He is usually described by the simple Heisenberg model with nearest neighbor interacion. Our conjecture is that hcp ^{3}He requires an advanced model with long rang interactions and multiple spin couplings. The recent susceptibility data of hcp ^{3}He below 100 μK seems to be better explained by such an advanced model.

There exists a widespread sentiment that magnetism of hcp ^{3}He is described by the nearest neighbor ferromagnetic Heisenberg model.[1),2)] Yet, the possibility of the existence of more long ranged interactions and of four spin interactions has not been excluded. The recent observtion of the magnetic susceptibility of hcp ^{3}He below 100 μK seems to be better explained by such an advanced model.[3)] On the basis of such model we give here theoretical formulae for some coefficients in the high temperature expansion scheme for hcp ^{3}He in the paramagnetic regime not very far from the ordered phase.

Solid ^{3}He is a unique quantum system, where 1) the zero point oscillations of atoms are quite large and 2) the quantum tunneling motions lead to the exchanges of atoms. The conventional theoretical analysis of experimental data of bcc ^{3}He resorts to the ring exchange model, in which the exchanges are limited to the triple and the four particle cyclic ones. Roger, Derlieu and Hetherington (RDH) argue that the ring exchange model can be justified by the WKB approximation for the exchange tunneling. On the basis of the same approximation they conclude that in the case of hcp ^{3}He only the triple exchange process (through the smallest ring) is important. If so, magnetism of hcp ^{3}He is described simply by the ideal Heisenberg model with the ferromagnetic nearest neighbor interaction. This argument does not seem to be warranted, however.

As pointed out in our previous paper[4] and also in the recent critical review of Cross and Fisher,[5] the exchange tunneling may possibly be an outcome of the hybridization between the tunneling motion and the zero point vibrations of the neighboring atoms surrounding the exchanging particles. If this is the case, the



(a) (b)

Fig. 1 Pictorial representations of the interactions. A basal plane is drawn. The vertices are the sites for ^{3}He atoms on this plane. White circles represent certain ^{3}He sites on the adjacent plane. Pair type interactions are defined by (a). Planar four spin interactions are defined by (b).

four particle exchanges as well as certain long ranged pair exchanges could be essential even in the hcp phase.

In this letter we consider the case that hcp ^{3}He is described by the Heisenberg Hamiltonian with the pair exchanges for the n.n. pairs (J_1, J_1'), for the second (J_2), for the third (J_3), for the 4th neighbor pairs (J_4, J_4'), and also with the three types of planar four spin exchanges(K_1, K_2, K_3). The geometry of each interactions are shown in Figs. 1(a), 1(b).

Just as in the analysis of bcc ^{3}He, exsistence of the four particle exchanges can be detected by certain experiments at high temperatures. In this letter, we consider e_2, θ, and B which are defined by

$$Cv = \frac{Ne_2}{4T^2} \tag{1}$$

and

$$x^{-1} = C^{-1}(T-\theta+\frac{B}{T}) \tag{2}$$

where we take k_B (Boltzmann' s constant)$=1$.

For hcp ^{3}He these coefficients are obtained immediately as

$$\begin{aligned} e_2 = {} & 9J_1^2+9J_1'^2+9J_2^2+3J_3^2+9J_4^2+18J_4'^2 \\ & +\frac{9}{4}(8+4\lambda_1+3\lambda_1^2)K_1^2 \\ & +\frac{9}{2}(8+4\lambda_2+3\lambda_2^2)K_2^2 \\ & +\frac{9}{4}(8+4\lambda_3+3\lambda_3^2)K_3^2 \end{aligned} \tag{3}$$

$$\theta = 3J_1+3J_1'+3J_2+J_3+3J_4+6J_4' \tag{4}$$

and

$$\begin{aligned} B = {} & 3J_1^2+3J_1'^2+3J_2^2+J_3^2+3J_4^2+6J_4'^2 \\ & -\frac{3}{4}\{(18+7\lambda_1)J_1K_1+(2+3\lambda_1)J_4K_1-(8+4\lambda_1+3\lambda_1^2)K_1^2\} \\ & -\frac{3}{2}\{(8+2\lambda_2)J_1K_2+(10+5\lambda_2)J_1'K_2+(2+3\lambda_2)J_4'K_2 \\ & -(8+4\lambda_2+3\lambda_2^2)K_2^2\}-\frac{3}{2}\{(4+\lambda_3)J_1K_3+(4+\lambda_3)J_1'K_3 \\ & +(2+3\lambda_2)J_2K_3-\frac{1}{2}(8+4\lambda_3+3\lambda_3^2)K_3^2\} \end{aligned} \tag{5}$$

It is true that the parameters $\lambda_i(i=1,2,3)$ are simply -1, if *only* cyclic exchange processes are allowed. However, they may in general take any value between -1 and 1.[6]

To clarify the issue, we introduce the dimensionless parameter $e_2^* = e_2/\theta^2$. If only the triple exchange is effective in hcp ^{3}He, we observe that

$$e_2^* = \frac{18J_1^2}{(6J_1)^2} = \frac{1}{2} \qquad (6)$$

On the other hand , if the inter-spin couplings are more complicated than the simple nearest neighbor type, we have $e_2^* > 1/2$. A more accurate estimate of e_2^* requires further experiments which should be performed with high priority. In the case of bcc ^{3}He, the nearest neighbor antiferromagnetic Heisenberg Hamiltonian gives $e_2^{*bcc} = 3/4$, while the RDH's two parameter model provides rather larger value $e_2^{*bcc} = 1.42$. The enhancement factor e_2^{*bcc} is then 1.89. This value seems to be constant, i.e., independent of the densities in the bcc phase. Considering this

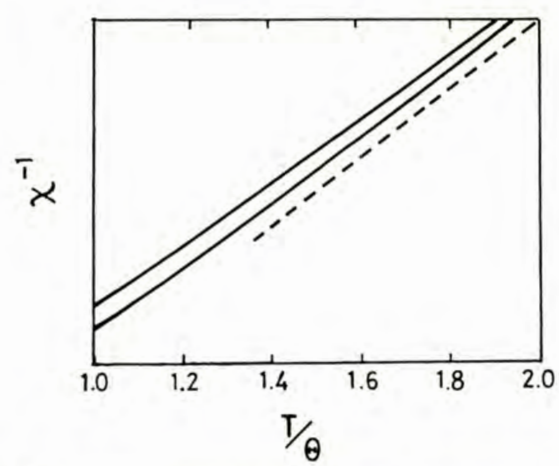


Fig. 2 The inverse susceptibility χ^{-1} versus temperature T. The upper solid line is the prediction of the nearest neighbor ferromagnetic Heisenberg model. The lower solid line is the result of the abvanced model with the following parameters; J1=J1' $=-2.71\,\mu$K, J2= J3= J4=J4' $=3.79\,\mu$K, K1=K2=K3$=-2.27\,\mu$K and $\lambda_i=-1(i=1,2,3)$. The value of B is appreciably smaller in the latter case than in the former. A broken line represents the Curie-Weiss law, i.e. B=0; the experimental data for hcp ^{3}He with the molar specific volume 19.0 cm^3/mol is also represented by the broken line.

fact, we can not exclude the possibility that the enhancement factor remains to be nearly the same even in the hcp phase. Although this has neither experimental nor theoretical justification, the conjecture could provide certain framework for the parameter adjustment, which could otherwise be arbitrary at this time. Thus, a speculative value $e_2^* = 0.95$ is obtained for hcp ^{3}He.

We observe also that the ISSP data on the susceptibility of hcp ^{3}He fit in remarkably well with the Curie–Weiss law even in the ultra-low temperature region, where the nearest neighbor Heisenberg ferromagnet is expected to deviate from the Curie–Weiss behavior. We expect, therefore, that B is much smaller than the value given by the Heisenberg ferromagnet with the nearest neighbor coupling. This is certainly possible if we include long range ferromagnetic interactions. We do not believe such simple long range interactions, however. In fact, crucial point of our hybridization model is that the model can predict the ferromagnetic Curie–Weiss constant ($\theta > 0$), even though J_1 and J_1' are antiferromagnetic. Although the negative J_1 and J_1' will act against the reduction of B, we can easily take parameter values which lead to a very small value of B. However, the large value of e_2^*, if we accept this condition, impose another restriction on the reduction of B. Yet, we can have a very small value of B, even if e_2^* takes such large value as $e_2^* = 0.95$. In fact, the Curie–Weiss behavior observed experimentally is explained by the appropriately chosen parameter values satisfying the conditions mentioned above. This is illustrated by Fig. 2.

We have reported theoretical analysis based on a speculative model. To determine the validity of the model, the specific heat, the pressure and the more precise susceptibility measurements in hcp ^{3}He should be carried out.

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